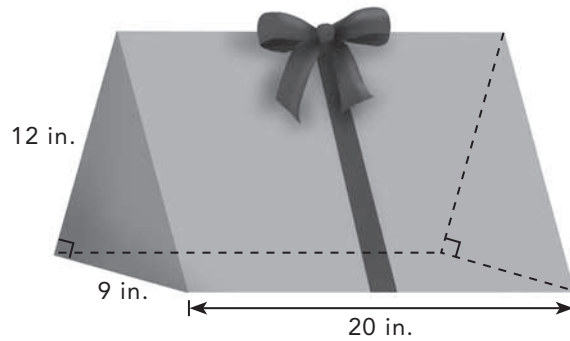## Lesson 12.3   Volume of Prisms

Solve. Show your work.

1. A cube has edges measuring 8 inches each. Find the volume of the cube.

2. A box is shaped like a rectangular prism. The box is 3.5 feet long, 1.8 feet wide, and 2 feet high. Find the volume of the box.

3. Find the volume of the gift box.

12 in.

9 in.

20 in.

4. The solid below is made of identical cubes. Each cube has an edge length of 3 inches. Find the volume of the solid.
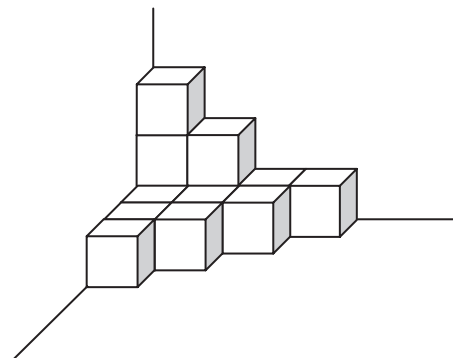
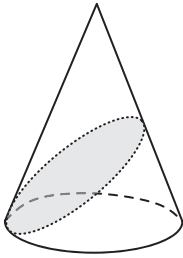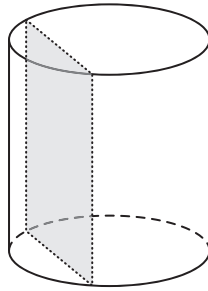**Tell whether slices parallel to each given slice will form uniform cross sections. If not, explain why not.**

5.

6.

7.
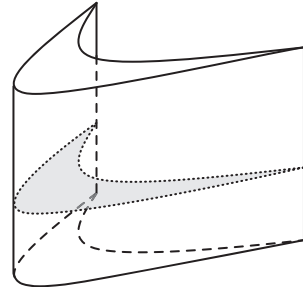
Solve. Show your work.

8. A cube has a volume of 512 cubic centimeters. Find the area of each face of the cube.

9. The volume of a rectangular prism with square bases is 5,880 cubic inches. It has a height of 30 inches. Find the side length of the square base.

10. A block of copper in the shape of a rectangular prism is 12 inches long, 6 inches wide, and 3 inches high. It is melted and recast into a cube. Find the edge length of the cube.
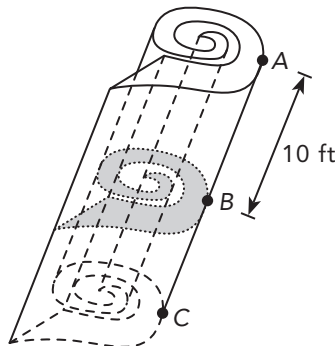
11. The bases of the prism shown are trapezoids. Find the volume of the prism.

3 in.

5 in.

28 in.

10 in.
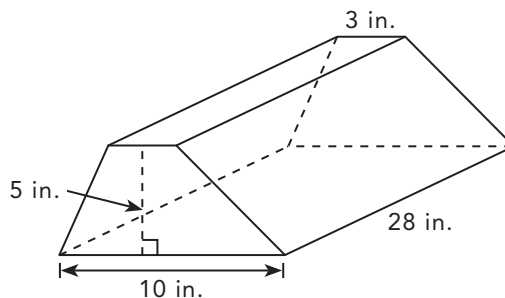
12. A cross section of the prism shown is parallel to a base. The area of the cross section is 78.5 square feet. The ratio of *AB* to *BC* is 5 : 4. The length of $\overline{AB}$ is 10 feet. Find the volume of the prism.

• A

10 ft

• B

• C

13. The solid is made by removing a smaller rectangular prism from a larger rectangular prism. Both prisms have square bases. The side length of the square base of the smaller prism is 4 centimeters. The side length of the square base of the larger prism is 12 centimeters. Find the volume of the solid.

12 cm

30 cm

12 cm
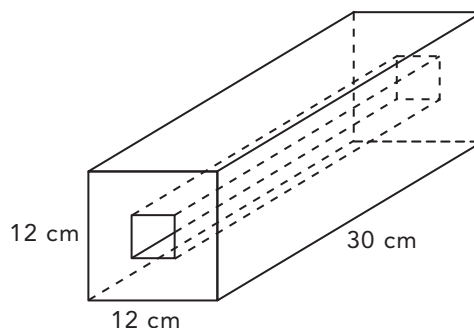
Lesson 12.2

1. Area of one square face
   $= 9 \cdot 9 = 81$ in.^{2}
   Surface area of cube
   $= 81 \cdot 6 = 486$ in.^{2}
   The surface area of the cube
   is 486 square inches.

2. Area of two rectangular bases
   $= 2 \cdot 20 \cdot 8 = 320$ ft^2
   Total area of the other four rectangular faces
   $= (20 + 8 + 20 + 8) \cdot 6$
   $= 336$ ft^2
   Surface area of rectangular prism
   $= 320 + 336 = 656$ ft^2
   The surface area of the rectangular prism is
   656 square feet.

3. Area of two triangular bases
   $= \left(\dfrac{1}{2} \cdot 4 \cdot 7\right) \cdot 2 = 28$ cm^2
   Total area of three rectangular faces
   $= (4 + 7.3 + 7.3) \cdot 12 = 223.2$ cm^2
   Surface area of prism
   $= 28 + 223.2 = 251.2$ cm^2
   The surface area of the prism is
   251.2 square centimeters.

4. Area of two triangular bases
   $= \left(\dfrac{1}{2} \cdot 16 \cdot 12\right) \cdot 2$
   $= 192$ in.^{2}
   Area of the other three rectangular faces
   $= (20 + 12 + 16) \cdot 4$
   $= 192$ in.^{2}
   Surface area of container
   $= 192 + 192 = 384$ in.^{2}
   The surface area of the container is
   384 square inches.

5. Area of two trapezoidal bases
   $= \dfrac{1}{2} \cdot 20 \cdot (10 + 17) \cdot 2 = 540$ cm^2
   Area of four rectangular faces
   $= (20 + 10 + 21.2 + 17) \cdot 24$
   $= 1,636.8$ cm^2
   Surface area of prism
   $= 540 + 1,636.8 = 2,176.8$ cm^2
   The surface area of the prism is
   2,176.8 square centimeters.

6. Area of large triangular base
   $= \dfrac{1}{2} \cdot 8 \cdot 6.9 = 27.6$ in.^{2}
   Area of small triangular base
   $= \dfrac{1}{2} \cdot 5 \cdot 4.3 = 10.75$ in.^{2}
   Area of three trapezoidal faces
   $= \dfrac{1}{2} \cdot 15 \cdot (8 + 5) \cdot 3 = 292.5$ in.^{2}

Surface area of solid
$= 27.6 + 10.75 + 292.5$
$= 330.85$ in.^{2}
The surface area of the solid is
330.85 square inches.

7. Area of one pentagonal base
   $= (120 \cdot 8) + \dfrac{1}{2} \cdot 120 \cdot (19 - 8)$
   $= 1,620$ ft^2
   Area of two pentagonal bases
   $= 1,620 \cdot 2 = 3,240$ ft^2
   Area of two rectangular faces
   $= 125 \cdot 8 \cdot 2 = 2,000$ ft^2
   Total area of walls to be painted
   $= (3,240 + 2,000) - 225 = 5,015$ ft^2
   The total area of the walls that need to be
   painted is 5,015 square feet.

8. a) Each base of the prism has $\dfrac{m}{2}$ sides.

   b) The prism has $1\dfrac{1}{2}m$ edges.

   c) The prism has $\left(2 + \dfrac{m}{2}\right)$ faces.

Lesson 12.3

1. Volume $= 8^3 = 512$ in.^{3}
   The volume of the cube is 512 cubic inches.

2. Volume $= 3.5 \cdot 1.8 \cdot 2 = 12.6$ ft^3
   The volume of the box is 12.6 cubic feet.

3. Area of triangular base
   $= \dfrac{1}{2} \cdot 9 \cdot 12 = 54$ in.^{2}
   Volume $= 54 \cdot 20 = 1,080$ in.^{3}
   The volume of the triangular prism is
   1,080 cubic inches.

4. Volume of each cube
   $= 3^3 = 27$ in.^{3}
   There are 13 cubes.
   Volume of solid $= 27 \cdot 13 = 351$ in.^{3}
   The volume of the solid is 351 cubic inches.

5. No. Ovals will have different dimensions with
   other cuts.

6. No. Rectangles will have different dimensions
   with other cuts.

7. Yes.

8. Edge length of cube $= \sqrt[3]{512} = 8$ cm
   Area of each face $= 8^2 = 64$ cm^2
   The area of each face of the cube is
   64 square centimeters.

9. Area of square base $= 5,880 \div 30 = 196$ in.^{2}
   Side length of square base $= \sqrt{196} = 14$ in.
   The side length of the square base is
   14 inches.

10. Volume of rectangular prism
$= 12 \cdot 6 \cdot 3 = 216$ in.³
Edge length of cube $= \sqrt[3]{216} = 6$ in.
The edge length of the cube is 6 inches.

11. Area of trapezoidal base
$= \dfrac{1}{2} \cdot 5 \cdot (3 + 10) = 32.5$ in.²
Volume of prism $= 32.5 \cdot 28 = 910$ in.³
The volume of the prism is 910 cubic inches.

12. Height of prism $= 10 \div 5 \cdot 9 = 18$ ft
Volume of prism $= 78.5 \cdot 18 = 1,413$ ft³
The volume of the prism is 1,413 cubic feet.

13. Volume of smaller prism
$= 4 \cdot 4 \cdot 30 = 480$ cm³
Volume of solid
$= (12 \cdot 12 \cdot 30) - 480$
$= 3,840$ cm³
The volume of the solid is
3,840 cubic centimeters.

Lesson 12.4

1. Area of two triangular bases
$= 2 \cdot \dfrac{1}{2} \cdot 6 \cdot 4 = 24$ ft²
Surface area $=$ (area of two triangular bases)
$\qquad\qquad + (5 + 5 + 6) \cdot h$
$\qquad 136 = 24 + 16 \cdot h$
$136 - 24 = 24 + 16h - 24$
$\qquad 112 = 16h$
$\qquad\quad 7 = h$
The height of the prism is 7 feet.

2. Height of empty portion of tank
$= 30 - 18 = 12$ cm
Volume of water needed
$= 60 \cdot 40 \cdot 12 = 28,800$ cm³
The volume of water needed is
28,800 cubic centimeters.

3. Volume of wall $= 450 \cdot 18 \cdot 108$
$\qquad\qquad\qquad = 874,800$ cm³
Number of bricks
$= 874,800 \div 972 = 900$
There are 900 bricks in the wall.

4. Volume of Box A
$= 18 \cdot 12 \cdot 10 = 2,160$ in.³
Volume of Box B
$= 2,160 \div 2 = 1,080$ in.³
Volume of Box B $= 72 \cdot h$
$\qquad\quad 1,080 = 72h$
$\qquad\qquad 15 = h$
The height of Box B is 15 inches.

5. a) Area of base
$= (20 \cdot 15) - (8 \cdot 7) = 244$ in.²
Volume $= 244 \cdot 10 = 2,440$ in.³
The volume of the prism is
2,440 cubic inches.

b) Area of two bases $= 2 \cdot 244 = 488$ in.²
Surface area
$= 488 + (20 + 15 + 20 + 7 + 7$
$\quad + 15) \cdot 10$
$= 1,328$ in.²
The surface area of the prism is
1,328 square inches.

6. a) Volume of water in tank
$= 30 \cdot 25 \cdot 6 = 4,500$ cm³
Volume of water in pail
$= 4,500 \div 9 \cdot 4 = 2,000$ cm³
Capacity of pail
$= 2,000 \div 4 \cdot 5 = 2,500$ cm³
The capacity of the pail is
2,500 cubic centimeters.

b) Volume of water needed to fill tank
$= 30 \cdot 25 \cdot (36 - 6)$
$= 22,500$ cm³
Minimum number of pails of water needed
$= 22,500 \div 2,500 = 9$
The minimum number of pails needed to
fill the tank is 9.

7. a) Surface area $= \dfrac{1}{2} \cdot 50 \cdot (1.5 + 8) \cdot 2 +$
$\qquad\qquad\qquad (1.5 + 50 + 8 + 50.4) \cdot x$
$\qquad 3,222.5 = 475 + 109.9x$
$3,222.5 - 475 = 109.9x + 475 - 475$
$\qquad 2747.5 = 109.9x$
$\qquad\qquad 25 = x$
The measure of x is 25 meters.

b) Area of each trapezoidal base
$= \dfrac{1}{2} \cdot 50 \cdot (1.5 + 8) = 237.5$ m³
Volume of pool
$= 237.5 \cdot 25 = 5,937.5$ m³
The volume of the pool is
5,937.5 cubic meters.

8. a) $\dfrac{2}{3} - \dfrac{1}{4} = \dfrac{5}{12}$
3,000 cubic inches of water fills $\dfrac{5}{12}$
of the tank.
Capacity of the tank $= 3,000 \div 5 \cdot 12$
$\qquad\qquad\qquad\qquad = 7,200$ in.³
The capacity of the tank is
7,200 cubic inches.

b) Area of square base $= 20^2 = 400$ in.²
Height of tank $= 7,200 \div 400 = 18$ in.
Height of the water $= \dfrac{2}{3} \cdot 18 = 12$ in.
The height of the water is 12 inches.